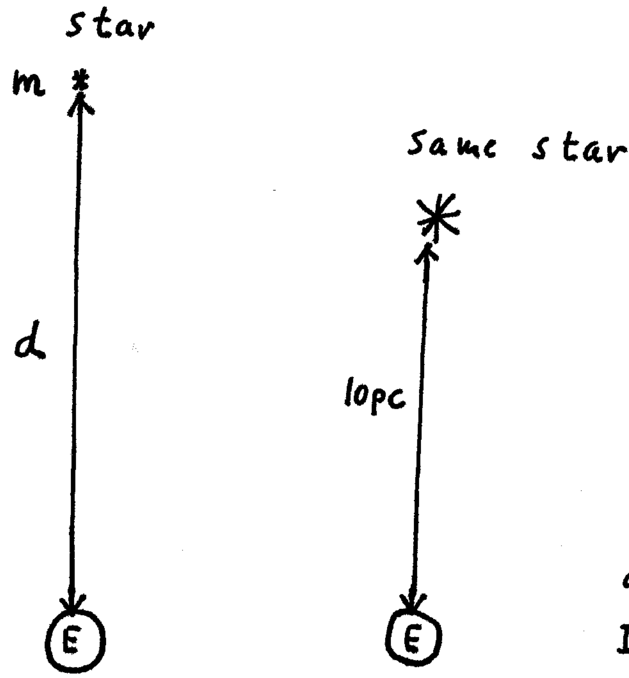
Relating absolute, M, and apparent, m, magnitudes

Consider a star of apparent magnitude m, at a distance d from an observer on the Earth. Its absolute magnitude is M; that is, this would be its magnitude were it at a distance of 10pc



It is awkward to work in magnitudes, since this scale is logarithmic: a change of 1 in magnitude represents a factor of 2·512 in brightness.

So, we will first consider the intensity of light reaching an observer from the star. Suppose that this intensity, at a distance d, is i and I when located at 10pc.

Using the inverse-square law, we know that if a star is twice as far away, its apparent brightness is reduced to a quarter. The apparent brightness changes to the inverse-square of the ratio of the distances. Hence, we can write:

$$\boxed{\frac{I}{i} = \left(\frac{d}{10}\right)^2} \Rightarrow$$

Taking logarithms (to base 10) of both sides gives:

$$\log_{10} I - \log_{10} i = 2\log_{10}\left(\frac{d}{10}\right) \quad \text{—— (1)}$$

From our earlier work on magnitudes, we have the following relationship between magnitude and intensity:

$$m = -2\cdot512 \log_{10} i + \text{constant} \quad \text{—— (2)}$$

$$M = -2\cdot512 \log_{10} I + \text{constant} \quad \text{—— (3)}$$

Subtracting (2) from (3), the constants cancel and we have:

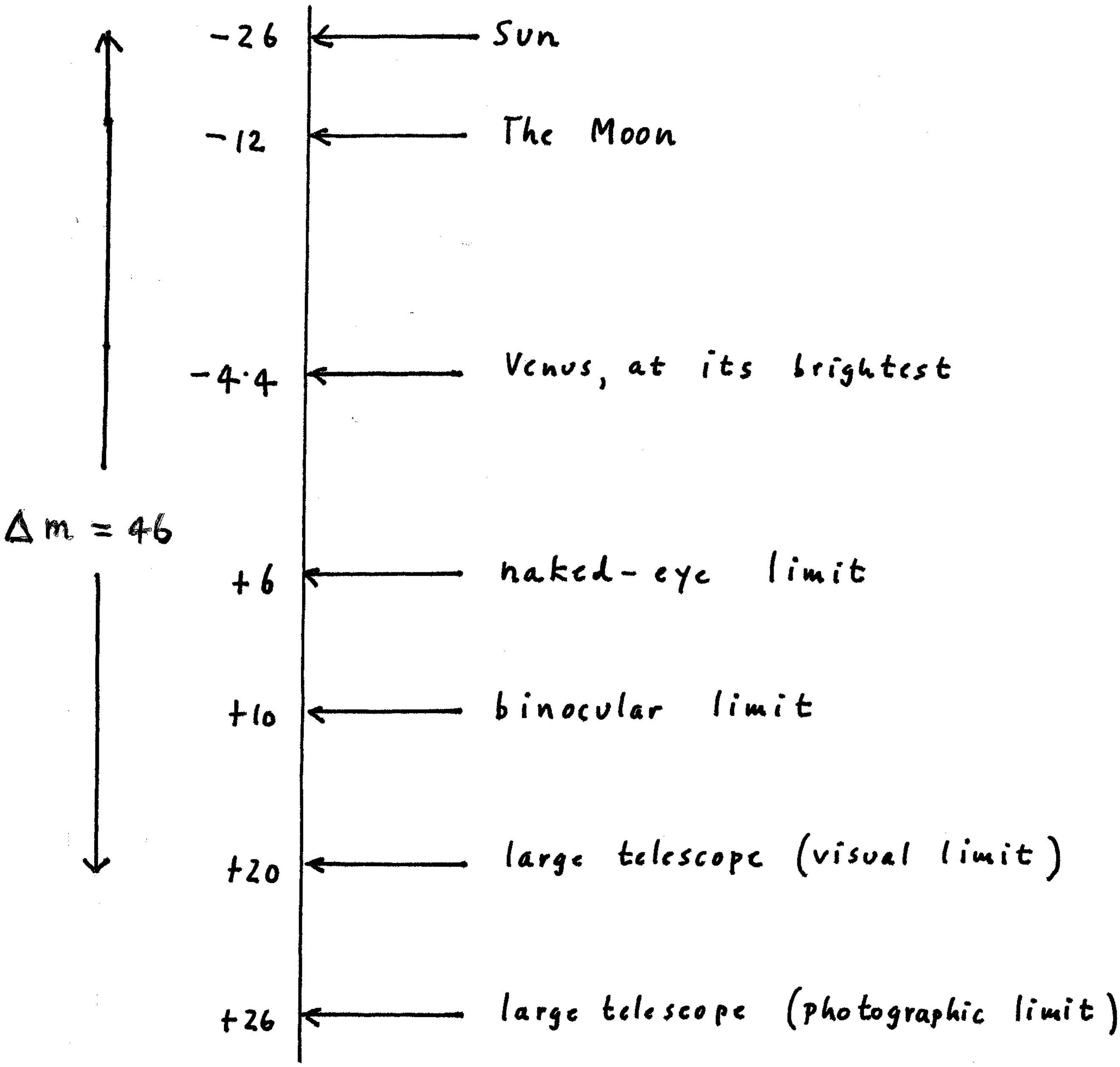
$$M - m = -2\cdot512\left(\log_{10} I - \log_{10} i\right)$$

Substituting for $\log_{10} I - \log_{10} i$ from (1) gives:

$$M - m = -2.512 \times 2 \log_{10}\left(\frac{d}{10}\right),$$

or

$$\boxed{m - M = 5.024 \log_{10}\left(\frac{d}{10}\right)}$$



$$\Rightarrow \Delta B = (2.512)^{46}$$

$$=$$

D.F.

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