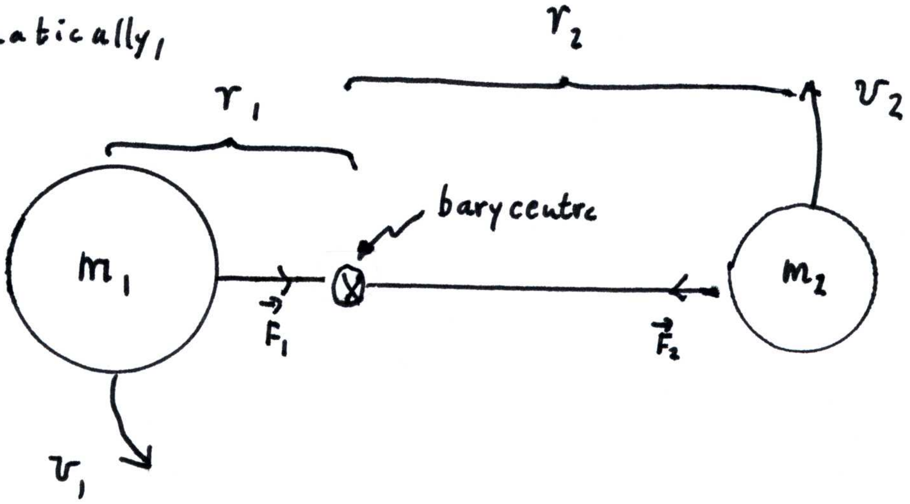
Binary Systems

If the masses are equal, the two objects will move in circles around a point mid-way along a line joining their centres (this point is known as the "barycentre"). They will (each) have identical orbital periods, speeds and radii.

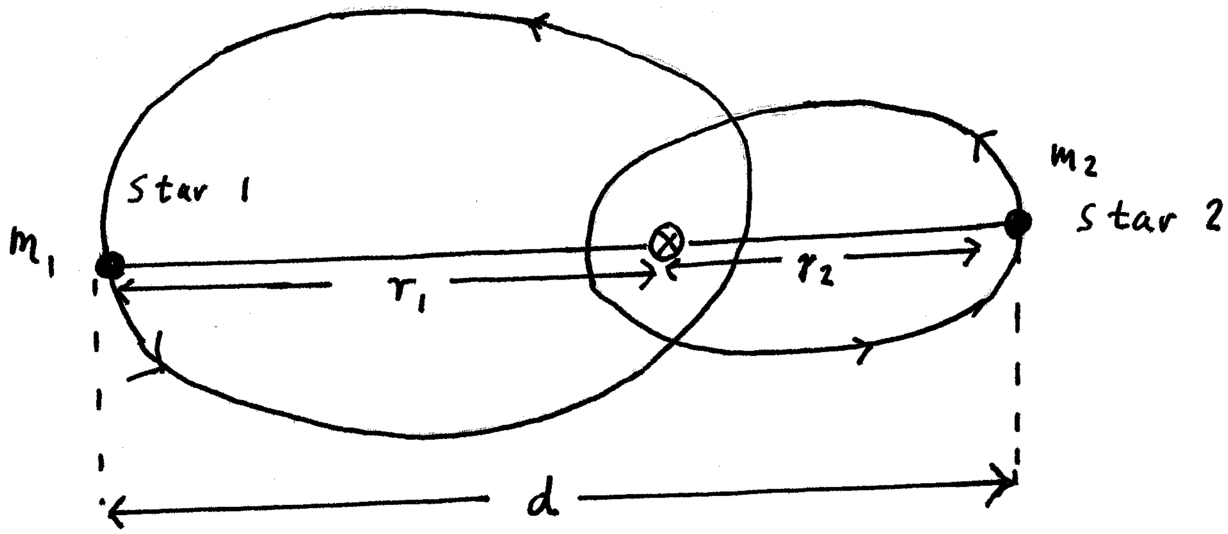
In the case of unequal masses, the centre of mass will be closer to the more massive object. For the barycentre to remain at rest, the two objects must always be on opposite sides of this point and so they both have the same orbital period. The smaller mass will have the larger orbital radius. Considering the Earth - Moon system, the mass of the Earth is $81 \times M_M$, so the barycentre is about one-third of the radius of the Earth, <u>below the surface of the Earth</u>. For the Earth - Sun system, $M_{\odot} = 300\,000\ M_E$, so the barycentre is practically at the centre of the Sun.

Schematically,



Clearly, $m_1 > m_2$

Their centripetal forces, $\vec{F}_1$ and $\vec{F}_2$, are both equal to the mutual gravitational force between them.



Two objects of masses m_1 and m_2 are separated by a distance d, experience a mutual, gravitational attracting force of magnitude $\vec{F}$, given by:

$$\vec{F} = \frac{G m_1 m_2}{d^2}$$

They orbit their common centre of mass, such that

$$m_1 r_1 = m_2 r_2 .$$

Since the two stars have the same orbital period, T, (the time taken to complete one revolution),

$$T = \frac{2\pi r_1}{v_1} = \frac{2\pi r_2}{v_2} \quad \text{— (1)}$$

The stars are separated by a distance $d = r_1 + r_2$. The magnitude of the attracting force between them is

$$\vec{F} = \frac{G m_1 m_2}{(r_1 + r_2)^2}$$

This must be the centripetal force for keeping each star in its orbit, so

$$\frac{m_1 v_1^2}{r_1} = \frac{G m_1 m_2}{(r_1 + r_2)^2} \quad \text{— (2)}$$

and

$$\frac{m_2 v_2^2}{r_2} = \frac{G m_1 m_2}{(r_1 + r_2)^2} \quad \text{—— (3)}$$

It is convenient to write equation (1), in terms of the angular velocity, ω.

That is,

$$\omega = \frac{v}{r},$$

giving

$$T = \frac{2\pi}{\omega}$$

Then rewriting equations (2) and (3), in terms of the angular velocity, ω, and cancelling m_1 and m_2, respectively, we get :

$$r_1 \omega^2 = \frac{G m_2}{(r_1 + r_2)^2} \quad \text{—— (4)}$$

and $$r_2 \omega^2 = \frac{G m_1}{(r_1 + r_2)^2} \quad \text{—— (5)}$$

Adding equations 4 and 5, using

$$d = r_1 + r_2$$ and

$$M = m_1 + m_2$$ (the total mass of the system),

we obtain

$$d\omega^2 = \frac{GM}{d^2}$$

Multiplying both sides by d^2 and dividing by G

$$M = \frac{d^3 \omega^2}{G} \quad \text{——— } 6$$

Rewriting ω in equation 6 by $\frac{2\pi}{T}$,

we obtain:

$$M = \frac{4\pi^2 d^3}{G T^2} \quad \text{——— } 7$$

or

$$T^2 = \frac{4\pi^2 d^3}{GM}$$

or

$$\boxed{\frac{T^2}{d^3} = \frac{4\pi^2}{GM}} \quad \text{——— } 8$$

Your old "friend",

K III.

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