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2018, January 22

A reminder of centripetal acceleration $\longrightarrow$ Kepler's Third Law of Planetary Motion

(or that of a secondary revolving around a primary)

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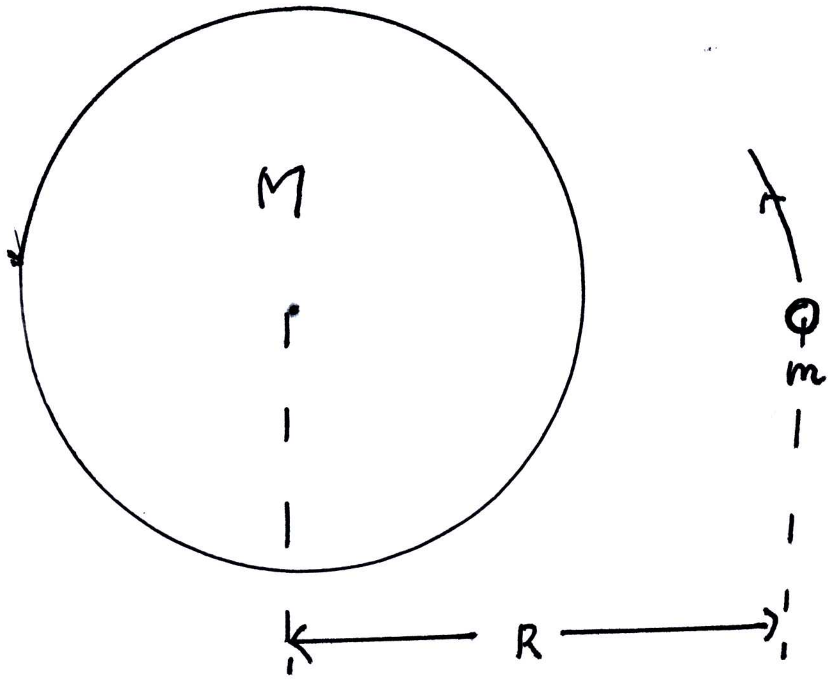

We know that the secondary body has an acceleration of $\dfrac{v^2}{R}$, directed towards the centre of the primary body.

It follows that the force required to maintain this acceleration = mass x acceleration.

That is,

$$\boxed{\vec{F} = m \cdot \left(\frac{v^2}{R}\right)} \quad \text{—— (1)}$$

The force is gravitational in origin.

Using Newton's Law of Universal Gravitation;

$$\boxed{\frac{G\,Mm}{R^2} = m\left(\frac{v^2}{R}\right)} \quad \text{—— (2)}$$

For the secondary body,

$$v = \frac{2\pi R}{T},$$

Where T is the orbital period.

$$\therefore \quad v^2 = \frac{4\pi^2 R^2}{T^2}$$

Simplifying, rearranging and substituting into, equation (2):

$$\frac{G\,M}{R^2} = \frac{4\pi^2 R^2}{T^2} \div R$$

That is,

$$\frac{GM}{R^2} = \frac{4\pi^2 R}{T^2}$$

Rearranging:

$$\frac{GM}{4\pi^2} = \frac{R^3}{T^2}$$

Inverting both sides

The square of the sidereal period of the secondary body is proportional to the cube of its mean distance from the primary body.

$$\boxed{\frac{T^2}{R^3} = \frac{4\pi^2}{GM}} \quad \text{—— (3)}$$

(a) For every particle comprising the rings of Saturn, orbiting at various values of R,

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM_S}.$$

(b) For the Jovian satellites,

$$\frac{T^2}{R^3} = \frac{4\pi^2}{GM_J}$$

(c) For all permanent members of the Solar System, including the comets,

$$\frac{T^2}{R^3} = \frac{4\pi^2}{G\,M_\odot}$$

$M_E = 6 \times 10^{24}\,kg$

$M_\odot = 2 \times 10^{30}\,kg$

$M_J = 318\,M_E$

$M_S = 95\,M_E$

From (3),

$T^2 \propto R^3$.

i.e.,

$T^2 = kR^3$,

where

$k = \dfrac{4\pi^2}{GM}$