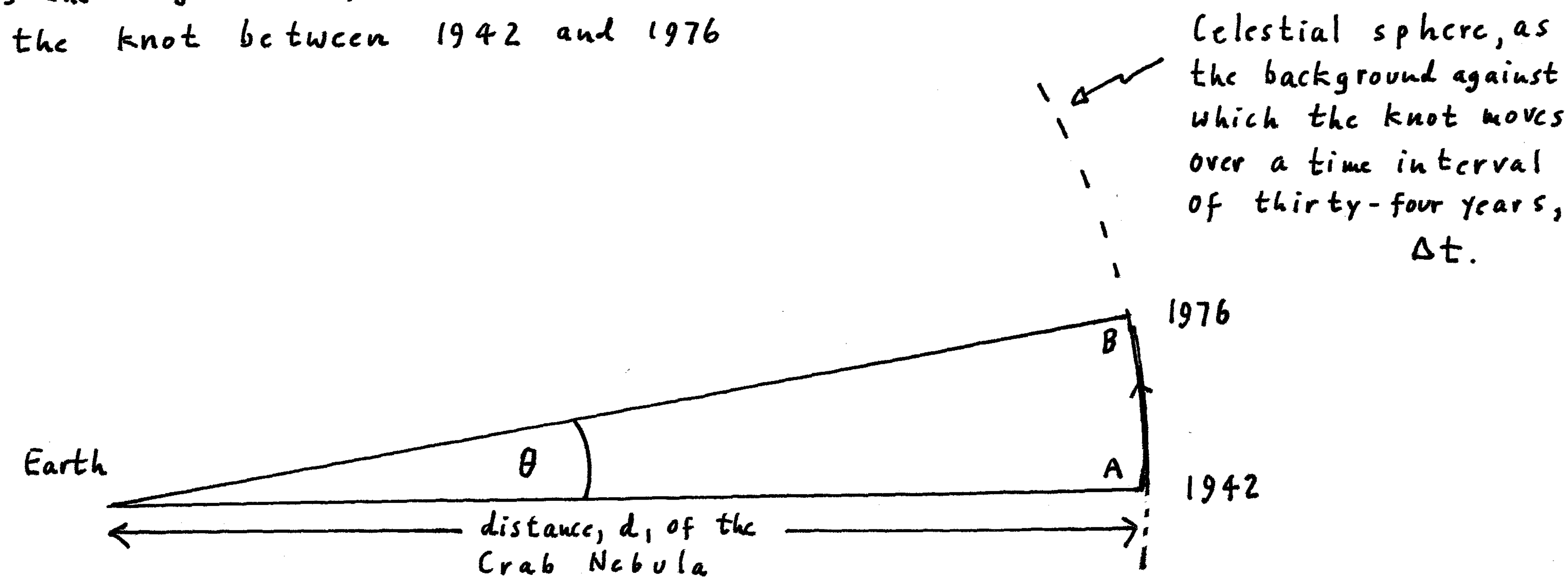
Measuring the distance to the Crab Nebula

θ is the angular displacement of the knot between 1942 and 1976



Then, arc length $\frac{\overrightarrow{AB}}{\text{total circumference of the celestial sphere at the distance, d, of the Crab Nebula}}$

is the same ratio as

$$\frac{\theta}{360^\circ}$$

(2)

That is,

$$\frac{\overrightarrow{AB}}{2\pi d} = \frac{\theta}{360^\circ}$$

Rearranging:

$$\theta = \frac{\overrightarrow{AB} \times 360}{2\pi d}$$

Now, θ is the total angular displacement of the knot over the specified time interval. If we require the <u>rate</u> of angular displacement, we must divide θ by this time interval. Of course, we must divide the r.h.s. by the same time interval, to preserve the equality.

Let this time interval be Δt.

$3600 = 1^\circ$

Then

$$\left(\frac{\theta}{\Delta t}\right) = \left(\frac{\overrightarrow{AB}}{\Delta t}\right) \times \frac{360^\circ}{2\pi d}$$

Study the r.h.s. of the above equation: only the arc distance, $\overrightarrow{AB}$, changes with respect to time; 360° and $2\pi d$ do not change.

The angular velocity of the knot, $\left(\frac{\theta}{\Delta t}\right)$, can be written as ω (arc seconds yr^{-1}) and $\left(\frac{\overrightarrow{AB}}{\Delta t}\right)$ is the spatial velocity of the knot (in $m\ s^{-1}$) which can be written as v.

$$\Rightarrow \quad \omega = v \times \frac{360 \times 3600}{2\pi d}$$

$$\therefore \quad \boxed{d = \frac{v \times 360 \times 3600}{2\pi \omega}}$$

<u>Note:</u> v has to be determined independently in order for d to be calculated. I made $\underline{v = 2.2 \times 10^5\ m\ s^{-1}}$

From an analysis, Using the Doppler Effect, of the spectrum.

DF
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