Nuclear reactions in main sequence stars

In a main sequence star, the nuclei present are almost entirely Hydrogen, i.e., protons, ${}^{1}_{1}H$ (also represented by the letter p) and Helium, ${}^{4}_{2}He$. Provided the temperature exceeds about 10^{7} K, protons can undergo a series of nuclear fusion reactions that produce Helium.

$$
\left.\begin{array}{l}
{}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu \\
{}^{1}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{2}He + \gamma \\
{}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + 2\,{}^{1}_{1}H
\end{array}\right\}
$$

${}^{2}_{1}H$ is a deuterium nucleus (also called a deuteron and is sometimes represented by ${}^{2}_{1}D$ or just D), consisting of a neutron and proton bound together, e^{+} is a positron (the antiparticle of the electron, also represented by ${}^{0}_{1}e$) and ν is a neutrino.

The above three equations make up the "pp I chain" ("pee-pee-one") pp for proton-proton, which converts Hydrogen to Helium. Notice that while six protons enter the chain via the first two equations, the two released in the third are "recycled".

A further reaction occurs when the positrons encounter electrons and annihilate,

$$e^{+} + e^{-} \rightarrow \gamma + \gamma$$

The net effect of the pp I chain is thus

$$4\,{}^{1}_{1}H + 2e^{-} \rightarrow {}^{4}_{2}He + 2\nu + 6\gamma$$

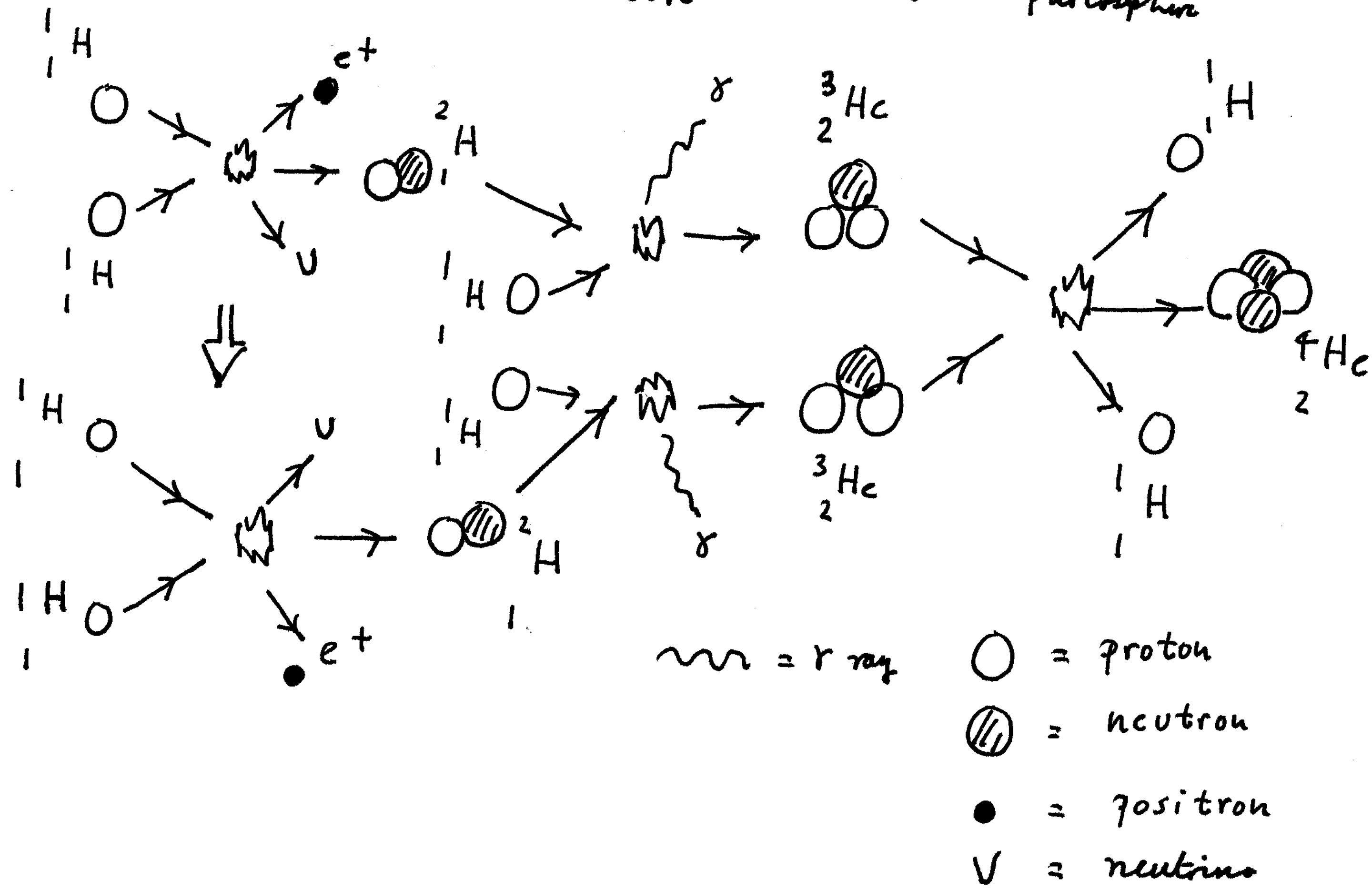
The pp I chain uses only Hydrogen as its input. However, since the star was "born" with Helium, contributing 8.5% of its nuclei (25% of its mass), the Helium produced in the last equation can react with existing ${}^{4}_{2}He$ nuclei, giving rise to slightly different chains of reactions, known as "ppII chains" and "pp III chains", which have exactly the same net effect as the "pp I chain". The "pp chains" are also known as the "pp cycles".

These pp chains are all quite slow. This is because the initial step (equation 1) is governed by the weak nuclear interaction, which has a short range. Even at temperatures above 10^{7} K, the probability of two protons approaching close enough to react is small. In our Sun, the average "survival" time of a proton before it reacts is 10^{10} years (sic).

Most of the subsequent reactions are governed by the

(2) strong nuclear interaction, which has a longer range than the weak interaction, so they are more rapid, but the overall rate of the pp cycles is governed by the first step. Although the Sun is producing enormous power, in nuclear reaction terms we should think of it as gently "simmering."

The temperature dependence of the first reaction is approximately proportional to T^4, so the overall rate of pp chains is sensitive to temperature : $T_{core} > 10^7$ K ; $T_{photosphere} = 6000$ K



The above schematic diagrams represent the first three reactions. These obey the <u>principle of conservation of electric charge and baryon number</u> (see later amplification)

<u>The law of conservation of energy</u> is also obeyed.
Remember that particles at rest have a certain amount of energy, called the <u>rest energy</u>, given by $\Delta E = \Delta m \cdot c^2$

∴ It is possible to estimate the amount of radiant energy ultimately released (as γ rays) by each occurrence

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of the pp I chain. In order to make this estimate, we shall assume that, once all the supplementary reactions are taken into account, there is no significant change in the k.e. of the particles present before and after each occurrence of the pp I chain. Neglect the 2% energy of the two neutrinos.
With these assumptions, we have to consider only the rest energies involved in the pp I chain — though we must remember to add the contribution arising from the annihilation of the two positrons with two of the Sun's electrons to produce yet more radiant energy. Thus:

radiant energy eventually resulting from each occurrence of the pp I chain

$$= c^2\left[\left(4 \times \text{rest energy of } {}^1_1H\right) - \left(\text{rest energy of } {}^4_2He + 2e^+\right)\right] + \left(\text{rest energy of } 2e^+ + 2e^-\right)$$

So, here is your chance.

<u>Question</u> The mass of the Helium nucleus, ${}^4_2He$, is $6{\cdot}645 \times 10^{-27}$ kg. That of the Hydrogen nucleus, ${}^1_1H$, is $1{\cdot}673 \times 10^{-27}$ kg and that of each positron or electron is $9{\cdot}110 \times 10^{-31}$ kg.

(a) Use this information to find the radiant energy eventually resulting from each occurrence of the pp I chain.

(b) Use your answer to (a), together with the solar Luminosity of $3{\cdot}84 \times 10^{26}$ J s^{-1} (W), to estimate the rate at which the pp I chain occurs.

(c) J F^2 ! Use your answer to (b) to estimate the mass of Hydrogen consumed yr^{-1} by the pp I chain.

Answers:

(a) The radiant energy eventually resulting from each occurrence of the pp I chain is given by the expression on side 3

$$= 9.00 \times 10^{16}\ \mathrm{m^2 s^{-2}} \left\{ 4 \times 1.673 - 6.645 + 2 \times 0.001 \right\} \times 10^{-27}\ \mathrm{kg}$$

$$= \underline{4.41 \times 10^{-12}\ \mathrm{J}}$$

(b) If we assume (somewhat incorrectly, Gentlemen) that the Solar Luminosity is provided by the pp I chain and its supplementary reactions, then, the number of times s^{-1} that the chain is completed is given by:

$$\frac{3.84 \times 10^{26}\ \mathrm{J\,s^{-1}}}{4.41 \times 10^{-12}\ \mathrm{J}}$$

$$= \underline{8.71 \times 10^{37}\ \mathrm{s^{-1}}}$$ (Not a bad answer, eh?)

(c) Each time the chain is completed, four Hydrogen nuclei are consumed (and one Helium nucleus produced)

Since the mass of a Hydrogen nucleus is 1.673×10^{-27} kg, it follows that the rate of Hydrogen consumption is roughly

$$4 \times 8.71 \times 10^{37}\ \mathrm{s^{-1}} \times 1.673 \times 10^{-27}\ \mathrm{kg}$$

$$= \underline{5.83 \times 10^{11}\ \mathrm{kg\,s^{-1}}}$$

Now, the number of seconds in a year is 3.16×10^{7} (everybody knows this!)

So, the annual consumption of Hydrogen is

$$3.16 \times 10^{7}\ \mathrm{s\,yr^{-1}} \times 5.83 \times 10^{11}\ \mathrm{kg\,s^{-1}}$$

$$= \underline{1.84 \times 10^{19}\ \mathrm{kg\,yr^{-1}}}$$

This is about three millionths of the mass of the Earth per year and about one part in 10^{11} of the mass of the Sun per year.

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